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Squaring the Circle

In the 1913 edition of the Journal of the Indian Mathematical Society, a proof of how to square a circle, or finding a square whose area is approximately the area of the circle. The late Ramanujan came up with this proof, and started off by saying that given a circle with center O, let PQR be in the circle with diameter PR. The overall concept of this proof is finding some constant that is very close to the length of the diameter, but when constructing a square by using that constant (which is RD in the case of this diagram), the square is very close to the circle in terms of size.

Say the circle has a diameter of 2. If point T is the point of trisection of OR closer to R, then the altitude QT can be drawn from point Q of triangle PQR to point T. The radius is 1, making TR become 1/3 and OT become 2/3. PO, the other radius, is also 1. Since the altitude drawn to the hypotenuse of a right triangle is the mean proportional between both segments of the hypotenuse, QT2 = . Looking at the diagram, a chord RS should be drawn such that RS is equal to the length of QT and that S is between points Q and R. This makes RS2=.

Angle PSR is a right angle, making triangle PSR a right triangle. Applying Pythagoras's Theorem, PS2 + RS2 = PR2. By substitution, PS2 + = 22. Solving for PS2 results in . Next, segments OM and TN should be drawn parallel to chord RS. By the side splitter theorem, triangle PMO is similar to triangle PSR, so =. This is equivalent to =. Substituting results in =. Solving for PM2 results in .

After that, chord PK should be placed such that PK=PM (which also means that PK2 = PM2=). Drawing from point P to point L such that line PL is tangent to line PR and equal to the length of MN, the length of PL can be found (PL2=MN2). Line PS and line PR are divided into proportional segments by the side splitter theorem. Using the proportion, =, and substituting PM2 for MN2, MN2=.

Drawing in segment RK and applying Pythagoras's theorem to right triangle PKR, RK2 + PK2=PR2. RK2 + = 22. Solving for RK2 yields . Drawing in RL2 and applying Pythagoras's theorem to right triangle RPL, RL2=PR2 + . Substituting PR2 for 22 and solving for RL2 results in. Place point C on line RH such that RC=RH=. Point D shall be placed at the intersection of line PK and RL, which makes segment RC parallel to line LK. Using the side splitter theorem in triangle LRK to form two similar right triangles, RDC and RLK, =. Substitution results in =. Solving for RD2 ends up as The constant mentioned in the first paragraph, RD, is actually about times the length of the radius.

Ramanujan stated at the end of his proof that if the area of the circle was 140,000 square miles, then the approximation for RD is greater than the true value of RD by about an inch. The area of the circle is pi\*radius2. Solving for the radius results in it being 119.100653 inches long... Since RD is approximated to be \* the radius, RD is 374.1657... miles long, or about 26131736.3 inches (Note that some calculators have a limited number of digits it can calculated up to, so results may vary). The ideal answer of RD is the square root of pi. If RD = the radius \* the square root of pi, RD is about 26131735.19 inches. The approximation of RD is greater than the ideal value of RD by about 1.11 inches. Ramanujan's method of squaring the circle is indeed, quite accurate.